

Finalists' Round

1. The sums of three whole numbers taken in pairs are 24, 34, and 38. What is the middle number?

- a. 14 b. 10 c. 7 d. 5 e. 24

2. Which of the following numbers is a perfect square?

- a. $\frac{14!15!}{2}$ b. $\frac{15!16!}{2}$ c. $\frac{16!17!}{2}$ d. $\frac{17!18!}{2}$ e. $\frac{18!19!}{2}$

3. The hundreds digit of a three-digit number is 3 more than the units digit. The digits of the three-digit number are reversed, and the result is subtracted from the original three-digit number. What is the hundreds digit of the result?

- a. 0 b. 2 c. 4 d. 6 e. 8

4. Johann has ~~64~~ fair coins. He flips all the coins. Any coin that lands on tails is tossed again. Coins that land on tails on the second toss are tossed a third time. Next Johann decides to flip all the coins which have landed on heads. What is the expected number of coins that are now tails?

5. A certain calculator has only two keys [+1] and [x2]. When you press one of the keys, the calculator automatically displays the result. For instance, if the calculator originally displayed "9" and you pressed [+1], it would display "10." If you then pressed [x2], it would display "20." Starting with the display "1," what is the fewest number of keystrokes you would need to reach "400"?

6. Let a , b , c , and d be real numbers with $|a-b| = 3$, $|b-c| = 4$, and $|c-d| = 5$. What is the sum of all possible values of $|a-d|$?

7. Real numbers x and y satisfy the equation $x^2 + y^2 = 10x - 6y - 34$. What is $x+y$?

8. How many positive integers N less than **200** are there such that the equation $x^{\lfloor x \rfloor} = N$ has a solution for x ? (The notation $\lfloor x \rfloor$ denotes the greatest integer that is less than or equal to x .)

9. Mr. Daly offers two of his students, Jeffrey and Eliza, an opportunity to earn extra credit. He gives each of them two worksheets: one on Math and one on Physics. Since Math is by far the more complex subject, the worksheet is worth 10 extra credit points, while the Physics worksheet is worth 3 extra credit points. However, since Mr. Daly is a clever and diabolical teacher, he gives them one condition: if both of them do the worksheet on Math, neither of them get any points. Jeffrey and Eliza planned to discuss who would do which extra credit worksheet after school, but Jeffrey forgot about his Academic Team practice and didn't meet Eliza, leaving her to figure out what to do on her own.

Eliza knows Jeffrey is a chronic overachiever, and knows Jeffrey will choose the Math worksheet 60 percent of the time. Based on this percentage, which worksheet should Eliza do to maximize her extra credit points? Assume that both of them, being awesome at math, can complete both worksheets accurately.